Exam Introduction to Logic (CS & MA)

Thursday 5 November 2015, 14 - 17 h.

- Write with a *blue or black pen* (so no pencil, no red pen).
- Write your *name and student number* on all pages of your work.
- Please fill out and hand in the anonymous evaluation form.
- Only hand in your definite answers. You can take the exam questions and any drafts home.
- With the regular exercises, you can earn 100 points. With the bonus exercise, you can earn 10 extra points.
  The exam grade is the number of points you earned divided by 10, with a maximum of 10. The final grade F is computed as

  $$F = 0.08 \cdot H_1 + 0.16 \cdot H_2 + 0.16 \cdot M + 0.60 \cdot E.$$

  Here H_1 is the grade for homework assignment 1, H_2 the grade for homework assignment 2, M the midterm exam grade, and E the grade for this exam.

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1. (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Do not forget to provide the translation key.

   (a) If Aida is not away, she will come unless it rains.

   (b) Although Serge neither feels ill nor avoids social contacts, he leaves his room only if he is hungry.

2. (15 points) Translate the following sentences to *first-order logic*. The domain of discourse is the set of students. Use the following translation key:

   v: Vera
   j: John
   m(x): the mentor of x
   A(x,y): x admires y

   (a) There are at least two students who have Vera as their mentor and admire her.

   (b) Everyone whose mentor is John, is the mentor of Vera or admires her.

   (c) Nobody admires all students that admire their mentor.

3. (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers.

   (a) Are the following two sentences tautologically equivalent?

      i. $\neg A \leftrightarrow (\neg B \rightarrow C)$

      ii. $(\neg A \wedge (\neg C \rightarrow B)) \vee (A \wedge \neg C \wedge \neg B)$

   (b) Is $((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow B) \rightarrow A$ a tautology?

4. (20 points) Give formal proofs of the following inferences. Do not forget the justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.

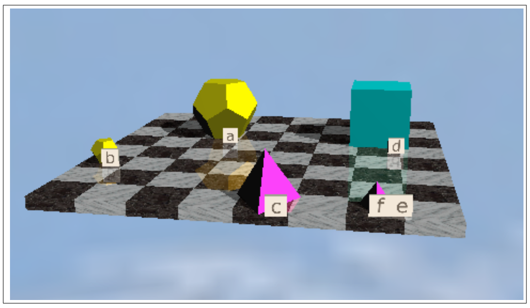
(a) $(A \to C) \lor (B \to C)$
— — —
$\neg C \to \neg(A \land B)$

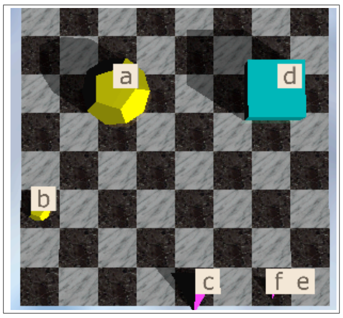
(b) $\exists x(A(x) \to B(x))$
— — —
$\neg\forall x(A(x) \land \neg B(x))$

(c) $\forall x(a = x \lor b = x)$
— — —
$(A(a) \land A(b)) \to \forall y A(y)$

(d) $\neg\forall x A(x)$
— — —
$\exists x \neg A(x)$

5. (15 points)





In the world displayed above a and d are large, c is medium and the other objects are small.

(a) In the world displayed above there is exactly one cube. Express this with one sentence in the language of Tarski's World. The sentence should be true in all worlds with exactly one cube, and false in all other worlds.

(b) Indicate of the sentence below, whether it is true or false in the world displayed above. Explain your answer.

$$\forall x \exists y(\mathsf{SameSize}(x, y) \land \neg\exists z\ \mathsf{LeftOf}(z, y))$$

(c) Indicate how the sentence below can be made true by removing one object from the world displayed above. Explain your answer.

$$\forall x(\exists y(\mathsf{SameCol}(x, y) \land x \neq y) \to \exists y \exists z \mathsf{Between}(x, y, z))$$

6. (15 points) Let a model $\mathfrak{M}$ with domain $D = \mathfrak{M}(\forall) = \{1, 2, 3\}$ be given such that

$$\mathfrak{M}(a) = 1 \quad \mathfrak{M}(b) = 3 \quad \mathfrak{M}(c) = 2$$
$$\mathfrak{M}(P) = \{1, 3\} \quad \mathfrak{M}(Q) = \{2, 3\}$$
$$\mathfrak{M}(R) = \{\langle 1, 3\rangle, \langle 2, 2\rangle, \langle 2, 1\rangle, \langle 3, 3\rangle\}$$

Let h be an assignment such that $h(x) = 2$, $h(y) = 3$, and $h(z) = 2$. Evaluate the following statements. Follow the definition of satisfaction (truth definition) step by step.

(a) $\mathfrak{M} \models P(a) \to (R(y, x) \lor Q(b))[h]$

(b) $\mathfrak{M} \models \forall x(P(x) \to R(z, x))[h]$

(c) $\mathfrak{M} \models \forall x \exists y R(x, y)[h]$

7. (15 points)

(a) Provide a conjunctive normal form (CNF) of the sentence $\neg((\neg A \land B) \lor C) \lor (A \land \neg D)$. Show all intermediate steps.

(b) Provide a Skolem normal form of the sentence $\exists x \forall y R(x, y) \to \forall x \exists z Q(x, z)$. Show all intermediate steps.

(c) Check the satisfiability of the Horn sentence below using the Horn algorithm.

$$A \land ((A \land B \land E) \to \bot) \land E \land (E \to D) \land (B \to A) \land ((D \land E) \to B)$$

8. (Bonus exercise: 10 points)
Give a formal proof for the inference

$\forall x \exists y(P(x) \land Q(y))$

$\exists y \forall x(P(x) \land Q(y))$

Do not forget to provide justifications. You may only use the Introduction and Elimination rules and the Reiteration rule.